**Homework 14**

**P12.1.9** The switch in Figure P12.1.9 is opened at *t* = 0 after being closed for a long time. (a) Determine *iL*, *vC*, *dvC*/*dt*, and *diL*/*dt* just after the switch is opened. (b) Is the circuit reducible to a prototypical series *LCR* circuit or a parallel *GCL* circuit?

**Solution:** At *t* = 0-, *iC* = 0 and the 3 mA divides between the 3 kΩ and 6 kΩ resistors, so that *IL*0 = 3×3/9 = 1 mA; *VC*0 = 6×1 = 6 V. These do not change on switching. At *t* = 0+, 2*iC* + *IL*0 = 0, so that *iC* = - *IL*0/2 = -0.5 mA = *C*, and **

-1000 V/s. From KVL, *VC*0 + 1×*iC* = 6*IL*0 + *vL*. Hence, *vL* = 6 – 0.5 – 6 = -0.5 V = *L*, and **=** = -5 A/s.

**P12.2.12** The switch in Figure P12.2.12, is moved to position ‘b’ at *t* = 0 after being in position a for a long time. (a) Choose *R* so that the response is critically damped; (b) determine the initial values *vC*(0+), *iL*(0+), and *vL*(0+); (c) determine *vC*(*t*)and *iL*(*t*) for *t* ≥ 0+.

**Solution:** (a) For critical damping, , where *Rs* = *R* + 500 Ω;  Ω. Hence, *R* = 500 Ω.

(b) When the switch is in position a, *iL* is 100 μA and *vC* is (100 μA)×(500 Ω) = 50 mV. When the switch is moved to position b, *iL* and *vC* at *t* = 0+ remain the same. From KVL: 50 = 0.1×1000 + *vL*, so that *vL* = -50 mV.

As , the energy initially stored in the circuit is dissipated and the current and all voltages become zero.

(c) Since the circuit is critically damped, ; at *t* = 0+, *vC*(0+) = 50 mV; hence, *A* = 50 mV; *iL* = -*Cdv*C/*dt* at *t* = 0+ = 10-8(-*ω*0*A* + *B*), or

-10-4 = 10-8(-*ω*0*A* + *B*), or -104 = -2×105×50×10-3 + *B*, or -104 = -104 + *B*, so that *B* = 0. It follows that *vC*(*t*) = = mV.

 *Cdv*C/*dt*: nF×mV/μs = 10-9×10-3/10-6 = 10-6 → μA;

*iL*(*t*) = -10×(-50)×0.2 =  μA.

**P12.2.18** The switch In Figure P12.2.18, is moved to position ‘b’ at  after being in position ‘a’ for a long time. Determine (*t*) for *t* ≥ 0+.

**Solution:** *iL*(0-) =  = 10 mA downwards, and *vO*(0-) is -5 V.

After the switch is moved to position ‘b’, the circuit becomes a parallel circuit having *C* = 1μF, *L* = 4 H, and *RP* = 500 Ω. Hence,  krad/s, rad/s; V, where  krad/s,  krad/s and *t* is in ms.   mA, where if *vO* is in V, *t* is ms and *L* is in henries, *i* is in mA. At *t* = 0, *A + B* = -5

and  mA. This gives:  V and  V. It follows that  V.

**P12.2.21** The switch in

 Figure P12.2.21

 Is closed at

 *t* = 0 after being opened for a long time. Determine: (a) *iL*(0-), *iC*(0-), *iL*(0+), *iC*(0+); (b) *iL*(*t*) for *t* ≥ 0+, assuming *R* = 1 Ω, *L* = 1 H, and *C* = 1 F.

**Solution:** (a) At *t* = 0-, the capacitor behaves as an open circuit and the inductor as a short circuit; *vC*(0-) = 0, where *vC* is a voltage drop in the direction of *iC*. The current through *R* next to the switch is *I*1 directed upwards. It follows from KVL around the mesh of three *R*’s, that the voltage across the source is zero, so that *I*1 = 10 A and *IL*(0-) = 2I1 = 20 A.

 At *t* = 0+, *I*1 = 5 A, and 2*I*1 = 10 A. The current through the inductor and the voltage across the capacitor do not change. It follows that *iC* = -10 A.

(b) *ω*0 = 1 rad/s, and *α* = = 1/2*RC* = 0.5 rad/s. . As *t* → ∞, *iL*(∞) = 10 A. Hence, ; At *t* = 0+, 20 = *A* + 10, so that *A* = 10. At *t* = 0+, *vC* = *LdiL*/*dt* = *L*(-*αA* + *ωdB*) = 0; *B* = *αA*/*ωd* . Hence,  A.

**P12.3.12** The switch in Figure P12.3.12 is opened at *t* = 0 after being closed for a long time. Determine *iL*(*t*) and *iC*(*t*) for *t* ≥ 0+.

**Solution:** At *t* = 0-, the capacitor acts as an open circuit and the inductor as a short circuit, It follows that *iC*(0-) = 0, *vC*(0-) = 0, *iL*(0-) = 2 + 10/5 = 4 A. At *t* = 0+, *vC* and *iL* do not change; hence, *iC*(0+) = -2 A. As *t* → ∞, *iC* = 0 and *iL* = 2 A.

 *α* = 1/(2×4×12.5×10-3) = 10 rad/s,  rad/s. The circuit is therefore critically damped. It follows that  A; at *t* = 0+, 4 = *A* + 2, so that *A* = 2 A; at *t* = 0+, *vC* = *LdiL*/*dt* = -*ω*0*A* + *B* = 0, so that *B* = *ω*0*A =* 20 A/s. It follows that:   A.

 *vC*(*t*) = *LdiL*/*dt* =  V;

*iC*(*t*) = *CdvC*/*dt* = -12.5×10-3×160 A.

**P12.3.16** Both switches in Figure

P12.3.16 are

moved

at *t* =

0 after being in their initial positions

for a long time. Determine *vC*(*t*) and *iL*(*t*) for *t* ≥ 0+.

**Solution:** At *t* = 0-, the 30 A source establishes initial conditions in the circuit. From current division, *iL*(0-) = 10 A and the current through the 4 Ω resistor is 20 A, so that

*vC*(0-) = 80 V. These do not change at *t* = 0+. As *t* → ∞, the capacitor acts as an open circuit and the inductor as a short circuit, so that *iL*(∞) = 0. The 5 A current flows through the 8 Ω resistor, making *vC*(∞) = 40 V.

 When the 5 A source is set to zero, the circuit is a series circuit; *α* = *R*/2*L* = 8/20 = 0.4 rad/s;  rad/s; the responses are underdamped, and  rad/s.

  V; At *t* = 0+, 80 = *A* + 40, so that *A* = 40 V; at *t* = 0+, *iL* = -*CdvC*/*dt* = -*C*(-*αA* + *ωdB*) = 10; *ωdB* = -25 + 0.4×40 = -9, *B* =

-9/0.3 = -30. Hence, .

 *iL*(*t*) = -*CdvC*/*dt* 

 A.

**P12.3.17** The switch in Figure P12.3.17 is moved at *t* = 0 from position ‘a’ to position ‘b’ after being in position ‘a’ for a long time. Determine *v*C(*t*) and *iL*(*t*) for *t* ≥ 0+.

**Solution:** At *t* = 0-, the inductor acts as a short circuit and the capacitor as open circuit. *iL*(0-) = 6 A; The current in the upper 10 Ω resistor is 2 A, and the voltage across it is *vC*(0-) = 20 V. When the switch is moved, the source current is reduced to 3 A, so that *iL*(∞) = 3 A and *vC*(∞) = 10 V.

 When the current source is set to zero, circuit reduced to a series *RLC* circuit having *R* = 20/3 Ω. Hence, *α* = *R*/2*L* = 5 rad/s and  rad/s. The responses are overdamped, with  rad/s and  rad/s.

  V; *vC*(0-) = 20 = *A* + *B* + 10, which makes *A* + *B* = 10. At *t* = 0+, 3 = *iL*(0+) + *CdvC*/*dt*, so that (1/6) *dvC*/*dt* = 1, or, *A* + 9*B* = 18; this gives *A* = 9 and *B* = 1. It follows that:  V, *t* is in s.

  A, *t* is in s.

**P12.3.18** The switch in Figure P12.3.18 is opened at *t* = 0 after being closed for a long time. Determine *vC*(*t*) and *iL*(*t*) for *t* ≥ 0+.

**Solution:** At *t* = 0-, the inductor acts a s a short circuit and the capacitance as an open circuit. The resistance in series with *L* is (6||12) = 4 Ω. *iL*(0-) = (30 – 10)/4 = 5 A. From current division, the current in the 6 Ω resistor on the right is 5/3 A, so that *vC*(0-) = 10 + 6(5/3) = 20 V.

 As *t* → ∞, with the capacitor acting as an open circuit, *iL*(∞) = 0 and *vC*(∞) = 10 V. The circuit is a series circuit, with *R* = 4 Ω. Hence, *α* = *R*/2*L* = 4/0.8 = 5 rad/s and  rad/s. The responses are critically damped

  V; *vC*(0-) = 20 = *A* + *B* + 10, which makes *A* + *B* = 10. At *t* = 0+, *iL*(0+) = -*CdvC*/*dt*, so that -(0.1)(-5A + *B*) = 5, or, 5*A* + *B* = 50; this gives *A* = 10 and *B* = 0. It follows that:  V, , *t* is in s

  A, *t* is in s